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EFFECTS OF ENERGY DISSIPATION AND MELTING ON SHOCK COMPRESSION OF POROUS BODIES

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UDC 534.222.2

Shock waves are a phenomenon not only associated with the possibility of destructive action but utilizable in a purely constructive manner: for molding of materials, welding, restoration of the thermodynamic parameters of condensed materials [1], construction of materials with a given degree of porosity, and the use of porous materials as a shock-wave damper [2].

The questions associated with the propagation of strong shock waves (pressures greater than 50 GPa) in porous bodies have been worked out in great detail [1]. The phenomenon in shock waves of strength from 0.1 GPa to tens of gigapascals has not yet been investigated in such great detail. In particular, the questions of the energy dissipation mechanism upon shock compression of porous bodies have not been investigated in this range of pressures.

Experiments with porous samples [3, 4] show that viscoplastic flow of material in the shock wave occurs most strongly in the vicinity of strong inhomogeneities. The local temperatures in these regions noticeably exceed the average temperature in the shock wave. Analogous effects are observed in connection with the explosive squeezing of cylindrical shells, where melting and evaporation of the inner surfaces of the shells occurred [5].

The distribution of the internal energy in the vicinity of an inhomogeneity is derived in this paper, and the nature of the energy accumulation and the effects of melting accompanying shock compression of porous bodies are analyzed on the basis of this distribution. The effect of thermal expansion of the material is investigated, and the behavior of the anomalous adiabats of porous media is studied in the pressure range comparable with the strength of the material. The problems of the structure of the shock front are considered, and the dependence of the melt volume on the wave amplitude is analyzed.

We will consider shock waves whose amplitude is small in comparison with the compressibility of a rigid body K ($K \sim 50$ GPa) but sufficiently large that a viscoplastic flow occurs in the vicinities of the pores. In this pressure range the width of the shock front is much larger than the sizes of the inhomogeneities [6]; one can assume the rigid body to

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Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 131-142, January-February, 1982. Original article submitted October 29, 1980.

0021-8944/82/2301-0123$07.50

be incompressible, since the density of the medium varies mainly due to plastic inflow of the pores. We will investigate the behavior of material with pores on the basis of the spherical-cell model [6]. The volume deformations of the cells is characterized by the variation of the macroscopic porosity parameter α, which is equal to the ratio of the total volume of the medium to the volume of the rigid body. In this model $\alpha = b^3/(b^3 - a^3)$, where b and a the the instantaneous radii of the cell and the pore. Spherical symmetrical compression of the cell is described by the pressure equation and the conditions of viscoplastic flow

$$\rho_s \frac{dv}{dt} = \frac{\partial\sigma_r}{\partial r} + 2\frac{\sigma_r - \sigma_\theta}{r}; \tag{1}$$

$$\sigma_r - \sigma_\theta = Y + 2\eta\,(\partial v/\partial r - v/r) \tag{2}$$

with the boundary conditions $\sigma_r(a) = 0$ (we will assume that the amount of gas in the pores is insignificant). Here σ_r and $\sigma_\theta = \sigma_\varphi$ are components of the local stress tensor, v is the velocity of radial motion, and ρ_s = const, Y, and η are the density, yield point, and viscosity of the rigid body. The mass conservation condition of material with ρ_s = const is of the form $r^3 - r_0^3 = a^3 - a_0^3$ (the subscript "0" refers to the initial values of the quantities), from which an expression is obtained for the velocity

$$v = a_0^3\dot{\alpha}/[3\,(\alpha_0 - 1)\,r^2]. \tag{3}$$

The system of mechanical equations (1)-(3) has been solved [6], thus permitting determination of the nature of the motion of the medium in the vicinities of pores and finding a relation between the average values of the pressure and density of the medium.

Let us investigate the effects of local heating of the material which viscoplastic inflow of the pores will produce. The equation for the energy dissipated per unit volume ε_d is written in the Lagrangian variables r_0 and t as

$$\frac{\partial\varepsilon_d}{\partial t} = \frac{2\,(\sigma_r - \sigma_\theta)}{3}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right). \tag{4}$$

Substituting the relationships (2) and (3) expressed in terms of the variables r_0 and t into Eq. (4) and integrating (4) over the time, we obtain

$$\varepsilon_d(r_0, t) = -\frac{2Y}{3}\ln\left[1 - \frac{a_0^3\,(\alpha_0 - \alpha)}{r_0^3\,(\alpha_0 - 1)}\right] + \frac{4\eta a_0^6}{3}\int\limits_{-\infty}^{t}\frac{\dot{\alpha}^2 dt}{\left[r_0^3\,(\alpha_0 - 1) - a_0^3\,(\alpha_0 - \alpha)\right]^2}. \tag{5}$$

Here the first term determines the contribution to energy dissipation by plastic friction, and the second term determines that due to viscosity. The total increment to the internal energy ε of the element of the medium having the coordinate r_0 (processes are considered whose durations $t_d \ll t_T$ and t_T are the characteristic times of the thermal conductivity) also includes the kinetic energy of motion of the material towards the centers of the cavities

$$\varepsilon_k(r_0, t) = \frac{\rho_s a_0^6\dot{\alpha}^2}{18\,(\alpha_0 - 1)^{2/3}\left[r_0^3\,(\alpha_0 - 1) - a_0^3\,(\alpha_0 - \alpha)\right]^{4/3}}, \quad \varepsilon = \varepsilon_d + \varepsilon_k, \tag{6}$$

which also changes into heat eventually.

Averaging of the quantity ε over the volume of the solid material of a cell gives the variation of the average value of the internal energy $\mathscr{E}$ per unit volume of the rigid body. The increment of energy of a unit mass of the solid component $\mathscr{E}/\rho_s$ is equal to the corresponding value for a porous medium, since one can neglect the surface energy of the pores [2]. Averaging the relationships (5) and (6), we arrive at an expression for $\mathscr{E}$ namely,

$$\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2 + \mathscr{E}_3, \tag{7}$$

$$\mathscr{E}_1 = \frac{2Y}{3}\left\{\alpha_0 \ln\frac{\alpha_0}{\alpha} - (\alpha_0 - 1)\ln\frac{\alpha_0 - 1}{\alpha - 1} + (\alpha_0 - \alpha)\ln\frac{\alpha}{\alpha - 1}\right\},$$

$$\mathscr{E}_2 = \frac{4\eta}{3}\int\limits_{-\infty}^{t}\frac{\dot{\alpha}^2 dt}{\alpha\,(\alpha - 1)}, \quad \mathscr{E}_3 = \frac{\rho_s\, a_0^2\dot{\alpha}^2}{6\,(\alpha_0 - 1)^{2/3}}\left[\frac{1}{(\alpha - 1)^{1/3}} - \frac{1}{\alpha^{1/3}}\right],$$

where $\mathscr{E}_1$ and $\mathscr{E}_2$ are the average values of the dissipated energy due to plastic and viscous friction and $\mathscr{E}_3$ is the average kinetic energy of motion which arises upon collapse of the cavities.

The expression for the internal energy can also be represented in the form of the work expended in closing the pores. The distribution of microstresses in the vicinity of a pore can be found by substituting (2) and (3) into (1) and integrating Eq. (1) over the radius with the boundary condition taken into account. As a result we obtain

$$\sigma_r = 2Y \ln\frac{a}{r} + \frac{4\eta a_0^3\dot{\alpha}}{3(\alpha_0 - 1)}\left(a^{-3} - r^{-3}\right) + \frac{\rho_s a_0^3}{3(\alpha_0 - 1)}\left[\ddot{\alpha}\left(a^{-1} - r^{-1}\right) + \frac{\dot{\alpha}^2 a_0^3}{6(\alpha_0 - 1)}\left(r^{-4} - a^{-4}\right)\right]. \tag{8}$$

We find the stress on the cell surface, which plays the role of an external force, by substituting r = b into (8) and transforming the result to the form

$$-\sigma_r|_{r=b} = \frac{\rho_s a_0^2}{3(\alpha_0 - 1)^{2/3}}\left\{\ddot{\alpha}\left[\alpha^{-\frac{1}{3}} - (\alpha - 1)^{-\frac{1}{3}}\right] + \frac{\dot{\alpha}^2}{6}\left[(\alpha - 1)^{-\frac{4}{3}} - \alpha^{-\frac{4}{3}}\right]\right\} - \frac{4\eta\dot{\alpha}}{3\alpha(\alpha - 1)} + \frac{2Y}{3}\ln\frac{\alpha}{\alpha - 1}. \tag{9}$$

Calculating the work of these forces, we obtain the following relationship: $\mathscr{E} = \int\limits_{\alpha_0}^{\alpha} \sigma_r(b)\, d\alpha$, where $\mathscr{E}$ is defined by expression (7). In particular, it follows from this that the average value of the internal energy of a porous medium is not equal to the integral of the average pressure P over the volume. This result follows from the fact that the quantities P and $-\sigma_r(b)$ differ for a porous body in the dynamical case, as has been noted in [6].

The rate of deformation of the material α is determined by the conservation laws on the shock front, which result in the dependence

$$P - P_0 = \rho_s D^2(\alpha_0 - \alpha)/\alpha_0^2 \tag{10}$$

(D is the velocity of the shock wave, and P_0 is the amplitude of the elastic precursor), and by the equation of state, which determines the relationship between P, α, and the derivatives of α. Thus the investigation of a plane steady front with the dynamics of viscoplastic inflow of the pores taken into account [6] has permitted determining an explicit form of the dependence $\dot{\alpha}(\alpha)$ for the case $1 \ll \eta/(\alpha_0\sqrt{\rho_s Y} \equiv R$:

$$\dot{\alpha}(\alpha) = \frac{3\alpha(\alpha - 1)}{4\eta}\rho_s D^2\left[\frac{\alpha - \alpha_0}{\alpha_0^2} + \frac{2Y}{3\rho_s D^2}\ln\frac{\alpha(\alpha_0 - 1)}{\alpha_0(\alpha - 1)}\right]. \tag{11}$$

Switching in expression (7) from the variable t to the variable α (in view of the fact that $\alpha(t)$ is a monotonic function in the loading wave) and performing the integration with Eq. (11) taken into account, we obtain the quantity $\mathscr{E}_2$ in the form

$$\mathscr{E}_2 = \frac{\rho_s D^2}{2\alpha_0^2}(\alpha_0 - \alpha)^2 + \frac{2Y}{3}\left\{\alpha\ln\frac{\alpha(\alpha_0 - 1)}{\alpha_0(\alpha - 1)} + \ln\frac{\alpha - 1}{\alpha_0 - 1}\right\}. \tag{12}$$

One can neglect the kinetic energy in this approximation, since $\mathscr{E}_3 \sim R^{-2} \ll 1$. Thus the energy dissipated in the shock wave front is represented in the form of two terms which depend upon the porosity. We will analyze the relationship of these quantities as a function of the shock wave amplitude. At low pressures in the wave when the final value of the porosity is close to the initial value α_0, the expressions for $\mathscr{E}_1$ and $\mathscr{E}_2$ can be represented in the form

$$\mathscr{E}_1 = P_0(\alpha_0 - \alpha), \quad \mathscr{E}_2 = \rho_s\left(D^2 - D_0^2\right)(\alpha_0 - \alpha)^2/\left(2\alpha_0^2\right), \tag{13}$$

where $P_0 = (2Y/3)\ln[\alpha_0/(\alpha_0 - 1]$ and $D_0 = \{2Y\alpha_0/[3\rho_s(\alpha_0 - 1)]\}^{1/2}$ is the minimum velocity of shock waves in a porous medium [6]. It follows from (13) that $\mathscr{E}_1 \gg \mathscr{E}_2$.

One can simplify the relationships for $\mathscr{E}_1$ and $\mathscr{E}_2$ for a strong shock wave by taking account of the fact that the final porosity is close to unity. Bearing in mind that when $R \gg 1$ the pressure and porosity behind the wave front are related as follows

$$P = P_e(\alpha) = \frac{2Y}{3}\ln\frac{\alpha}{\alpha - 1}, \tag{14}$$

when $\dot{\alpha} = 0$, and eliminating the quantity D from (12) with the help of (10), we write the expression for $\mathscr{E}_1$ and $\mathscr{E}_2$ in the form

$$\mathscr{E}_1 = (2Y/3)[\alpha_0 \ln \alpha_0 - (\alpha_0 - 1) \ln (\alpha_0 - 1)], \quad \mathscr{E}_2 = (P - P_0)(\alpha_0 - 1)/2 - (2Y/3) \ln \alpha_0.$$

Thus in the range of pressures $P \gg Y$, in which the pores are practically completely chosen, the quantity $\mathscr{E}_2 \gg \mathscr{E}_1$, i.e., energy dissipation occurs mainly by virtue of viscous friction. This result also follows from the fact that in the P—α diagram the term $\mathscr{E}_2$ is determined by the area bounded by the Rayleigh line and the static curve $P_e(\alpha)$.

Adding the expressions for $\mathscr{E}_1$ (7) and $\mathscr{E}_2$ (12) and representing the quantity D in terms of the shock wave amplitude, we have the relationship

$$\mathscr{E} = \mathscr{E}_1 + \mathscr{E}_2 = (\alpha_0 - \alpha)(P + P_0)/2, \tag{15}$$

which is valid at all points of a steady wavefront. Thus in the final state behind the shock wave front Eq. (15) coincides with the Hugoniot adiabat of the medium described in the usual form $\mathscr{E}/\rho_s = (V - V_0)(P + P_0)/2$, where P and V are the final values of the pressure and specific volume.

When inertial effects are taken into account, the relationship (15) is not satisfied, generally speaking, since part of the energy of the wave changes into kinetic energy of vibrational motions of the medium around the cavities, which was not taken into account here. In this case a deviation of the dynamic compression curve from the static curve $P_e(\alpha)$ occurs, which is related to the influence of inertia effects [6].

It is evident from an analysis of the expression (5) for the dissipated energy that the highest temperatures occur near the surface pores.

Substituting $r_0 = a_0$ into (5) and switching to the variable α, we obtain

$$\varepsilon_d(a_0, \alpha) = \frac{2Y}{3} \ln \frac{\alpha_0 - 1}{\alpha - 1} + \frac{4\eta D}{a_0} \int\limits_{\alpha}^{\alpha_0} \frac{d\alpha}{d\tau}(\alpha') \frac{d\alpha'}{(\alpha' - 1)^2}, \quad \tau = \frac{tD}{a_0}. \tag{16}$$

Upon collapse of the cavities in a solid body the asymptotic dependence $d\alpha/d\tau \sim (\alpha - 1)^{1/6}$ is satisfied [6]. From this we find that as $\alpha \to 1$ the term associated with plasticity has a logarithmic singularity, and the viscous term in (16) tends to ∞ as $(\alpha - 1)^{-5/6}$. The energy accumulation associated with plasticity can be eliminated if one takes account of the fact that at the melting temperatures the material in the vicinity of a pore changes into the liquid state with Y = 0.

In the case of a large initial porosity the heating of the material in a shock wave is so intense that the effects of thermal expansion of the material become important. Thus for $\alpha_0 \gtrsim 2$ and pressures in the wave higher than several gigapascals the density of the material behind the shock front decreases due to thermal effects [1, 7]. One should investigate the behavior of a porous medium in this pressure range with the compressibility of the material taken into account. To do this we will make use of the Mie—Gruneisen equation of state of a solid body in the form

$$E = E_c + V^{\Gamma}(Q - Q_c)/\Gamma, \tag{17}$$

where E and E_c are the total and cold energies of unit mass of solid material, Q and V_s are the pressure and specific volume of the solid material, and Γ is the Gruneisen coefficient, which one can assume to be constant for pressures $Q \ll K$. Taking account of the fact that the variations of the volume and the cold component of the pressure Q_c in this range of pressures are connected by a linearly elastic relationship [1], we have

$$\begin{aligned} Q_c - Q_c(V_{s0}) &= K(V_{s0} - V_s)/V^s{}_0, \\ E_c - E_c(V_{s0}) &= K(V_{s0} - V_s)^2/(2V_{s0}). \end{aligned} \tag{18}$$

Here the energy of unit mass of porous material is equal to the energy of continuous material if one neglects the surface energy of the pores. We write the shock adiabat of the medium in the usual form

$$E - E_0 = (P + P_0)(V_0 - V)/2, \tag{19}$$

which is obtained from (15) for the case $R \gg 1$ on the basis of an analysis of dissipative processes in the shock wave. Adding the relationship (14), which gives a dependence between the pressure and the porosity behind the shock front, to Eqs. (17)-(19), and taking the relations $Q = P\alpha$ and $V = V_s\alpha$ into account, we obtain a system of equations which determines the shock adiabat of a porous material. The final state of the medium behind the shock front is expressed by the values E, P, V, α, Q, and V_s. Thus, solving the equations for P and $V_1 = V/V_0$, we obtain the shock adiabat of a porous material in the form

$$V_1^2\frac{\alpha_0 h}{\alpha^2} + V_1\left\{\frac{Ph+P_0}{K} - \frac{h+1}{\alpha} - \frac{(1-h)}{\alpha K}Q_{c0}\right\} + \frac{1}{\alpha_0} + \frac{(1-h)}{\alpha_0 K}Q_{c0} - \frac{P+P_0 h}{K} = 0, \qquad (20)$$

where $\alpha = \alpha(P)$ is determined by expression (14). The parameter $h = 1 + 2/\Gamma$ and Q_{c0} can be found from the condition $Q_{c0} = P_0\alpha_0 - E_{s0}\Gamma/V_{s0}$, in which $E_{T0} = E_{T0}(T_0, V_{s0})$ is the value of the thermal energy of the material at the initial temperature T_0 and volume V_{s0}.

One should choose the solution with the plus sign in front of the quadratic root from the two solutions of the quadratic equation in V_1 (20), since it passes through the initial point $V_1 = 1$ and $P = P_0$. At low pressures, when one can neglect the terms P/K and P_0/K, Eq. (20) changes to the form $V_1 = \alpha/\alpha_0$ (or $V = V_{s0}\alpha$), which does not depend on h, K, and Q_{c0}. It follows from this that a change in the volume is related only to a decrease in the pore volume and one can neglect the heated material.

In the other limiting case as $\alpha \to 1$ Eq. (20) is solved for P and takes the form

$$P = \{K[V_1(h+1) - V_1^2\alpha_0 h - \alpha_0^{-1}] + (1-h)(V_1 - \alpha_0^{-1})Q_{c0} + P_0(h - V_1)\}\{hV_1 - 1\}^{-1}. \qquad (21)$$

This expression is in agreement with the results of [1], in which the shock adiabat of a porous material is obtained with neglect of the strength of the medium Y. Since it follows from analysis of (21) for $V \approx V_{s0}$ ($V_1 \approx \alpha_0^{-1}$) that for $\alpha_0 > h$ the volume increases and for $\alpha_0 < h$ it decreases, then the shock adiabats P(V) for $\alpha_0 > h$ have a turning point $V \approx V_{s0}$, which separates the section of the curve in which the volume increases when the shock wave amplitude increases. Calculations of the shock adiabats of porous copper according to Eqs. (14) and (20) are presented in Fig. 1 which are compared with experimental results [7]. The following values of the parameters were used: $K = 1.2\cdot10^2$ GPa, $V_{s0} = 1.1\cdot10^{-4}$ m³/kg, $Y = 0.23$ GPa, $E = 7.7\cdot10^4$ J/kg, $\Gamma = 1.9$ [1, 8], and $\alpha_0 = 1.91$ and 2.98 (curves 1 and 2); 1' and 2' are experimental curves for the very same values of α_0. The deviation of the calculated curves from the experimental ones in the region $P \sim Y$ may be related to deformational strengthening of the material, which was not taken into account.

The regions of applicability of formulas (20) and (21) are limited to pressures of several tens of gigapascals, since for larger pressures the nonlinear nature of the dependence $Q_c(V_s)$ is important. If one replaces the approximate relationships (18) with more general ones, then Eq. (21) (for the case $\alpha \to 1$) changes into the equation derived in [1].

Let us consider shock waves whose intensity is sufficient that molten zones are formed near the pores (the conditions for the occurence of these regions will be derived below). We will write the equations for the shear stresses in the solid and liquid phases in the vicinity of a pore in the form

$$\begin{aligned} \sigma_r - \sigma_\theta &= Y + 2\eta(\partial v/\partial r - v/r), \quad r_* < r \leqslant b, \\ \sigma_r - \sigma_\theta &= 2\mu(\partial v/\partial r - v/r), \quad a \leqslant r < r_*, \end{aligned} \qquad (22)$$

where η and μ are the viscosity coefficients of the solid body and the melt and $r_* = r_*(t)$ is the radius of the molten zone, for whose determination it is necessary to know the temperature distribution near the pores. The normal components of the local stress tensor

$$\sigma_r(a) = 0, \; \sigma_r(r_*+) = \sigma_r(r_*-) \qquad (23)$$

are continuous on the pore surfaces and the interphase boundaries. One can integrate the equation of spherically symmetric motion of material towards the center of a pore under conditions (22) and (23) if one assumes that the solid body is incompressible (an expression for the radial velocity (3) is known in this case). By determining the stress distribution near a pore and averaging it over the cell volume, we obtain the following dependence of the average pressure in the medium on the porosity and the radius of the molten zone [10]:

$$P = \frac{\rho_s a_0^2}{(\alpha_0 - 1)^{2/3}}\left\{-A(\alpha)\frac{\ddot{\alpha}}{\alpha} + B(\alpha)\frac{\dot{\alpha}^2}{6\alpha}\right\} - \frac{4\dot{\alpha}}{3\alpha}\left[\frac{(\eta-\mu)(1-\delta)}{\delta+\alpha-1} + \frac{\mu}{\alpha-1}\right] + \frac{2Y}{3}\ln\frac{\alpha}{\delta+\alpha-1}, \qquad (24)$$

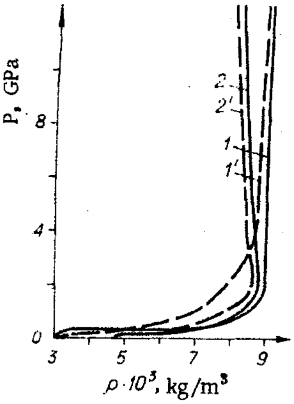

Fig. 1

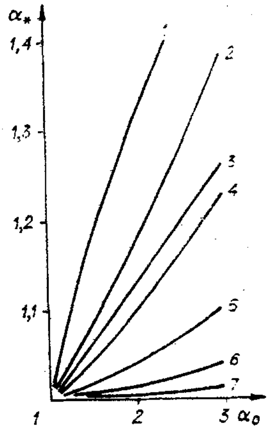

Fig. 2

$$A(\alpha)=\frac{1}{3(\alpha-1)^{1/3}}+\frac{(\alpha-1)^{2/3}-\alpha^{2/3}}{2},$$

$$B(\alpha)=\frac{1}{\alpha^{1/3}}-\frac{1}{(\alpha-1)^{1/3}}+\frac{1}{3(\alpha-1)^{4/3}},\ \delta=\frac{r_*^3-a^3}{b^3-a^3},$$

where δ is the relative fraction of molten material. For $\delta = 0$ expression (24) is similar to the relationship derived in [6, 9] in the absence of melting. The dynamical terms in the expression for P characterize the contributions due to inertia and viscosity, and the last term determines the equilibrium part of the total pressure.

Solving the system of equations (10) and (24), we obtain a second-order differential equation in the function $\alpha(\xi)$, where $\xi = (x - Dt)/a_0$, x is the coordinate in the propagation direction of the wave, and a_0 is the initial radius of the pores. For the function $g(\alpha) = d\alpha/d\xi$ this equation reduces to the first-order equation

$$A(\alpha)\frac{dg}{d\alpha}=B(\alpha)-\frac{g}{6}+(\alpha_0-1)^{\frac{2}{3}}\left\{\frac{4Rk}{3}\left[\frac{(1-n)(1-\delta)}{\delta+\alpha-1}+\frac{n}{\alpha-1}\right]-F(\alpha)\frac{\alpha(\alpha_0-1)^{2/3}}{g}\right\}, \tag{25}$$

$$F(\alpha)=\frac{2k^2}{3}\ln\frac{\alpha_0(\delta+\alpha-1)}{\alpha(\alpha_0-1)}+\frac{\alpha_0-\alpha}{\alpha_0^2},\ n=\frac{\mu}{\eta},$$

$$R=\eta/\left[a_0\sqrt{\rho_s Y}\right],\ k=\frac{1}{D}\sqrt{\frac{Y}{\rho_s}}$$

with the boundary condition $g(\alpha_0) = 0$. We make use of the distribution of the energy dissipated in the vicinity of a pore (Eq. (5)) to determine the unknown function $\delta = \delta(\alpha)$. The temperature on the boundary of the melt and the solid phase at $r = r_*$ is equal to the melting temperature T_*, and the increment to the thermal energy is $\varepsilon_* = \rho_s c_V(T_* - T_0) + \varepsilon_0$, where c_V is the specific heat, T_0 is the initial temperature, and ε_0 is the latent heat of fusion. Equating the expression for the energy dissipated near a pore to the value of ε_* and expressing r_* in terms of δ, we obtain the integral equation

$$E_0 = \ln\frac{\delta + \alpha_0 - 1}{\delta + \alpha - 1} + \frac{2R}{k}\int\limits_{\alpha}^{\alpha_0}\frac{g(\alpha', \delta')}{[\delta + \alpha' - 1]^2}\,d\alpha', \tag{26}$$

where $\delta' = \delta(\alpha')$ and $E_0 = 3\varepsilon_*/(2Y)$.

Thus, in the region of phase coexistence the behavior of a porous medium upon shock compression is determined by a system of differential and integral equations in the functions $g(\alpha)$ and $\delta(\alpha)$. The system has the solutions $\delta(\alpha) \geqslant 0$, which have physical meaning when $\alpha \leqslant \alpha_*$, where α_* is the critical value of the porosity corresponding to the origin of a melting zone on the pore surface. For $\alpha > \alpha_*$ the structure of the shock wave front is described by Eq. (25) with $\delta = 0$.

The solutions of Eqs. (25) and (26) which depend on the five dimensionless parameters (E_0, R, k, α_0, and n) describe the compression phase in the region $g \geqslant 0$, $\alpha_t \leqslant \alpha \leqslant \alpha_0$, where α_t is the first turning point, which is subject to determination in the course of solving the equations. Its value lies in the interval $1 \leqslant \alpha_t \leqslant \alpha_e$, where α_e is the porosity at the equilibrium position of the system ($d^2\alpha/d\xi^2 = g = 0$), which is specified by the equation

$$F(\alpha, \delta(\alpha), k, \alpha_0) = 0. \tag{27}$$

Investigation of the structure of the shock wave front [6, 9] shows that in the case $R \gg 1$ one can neglect the inertia terms in the expression for the pressure. At the same time Eq. (25) simplifies, which permits finding the function g in the form

$$g = \frac{3\alpha F(\alpha)(\delta + \alpha - 1)(\alpha - 1)}{4Rk[(1 - \delta)(\alpha - 1) + n\delta\alpha]}. \tag{28}$$

Eliminating g from (26), we arrive at a nonlinear integral equation for the function $\delta(\alpha)$.

The critical value of the porosity α_* at which melting starts in the wave is found by substituting $\delta = \delta' = 0$ and Eq. (28) into Eq. (26). Carrying out the integration, we obtain the following transcendental equation for determining α_*:

$$\frac{3k^2}{2\alpha_0^2}(\alpha_0 - \alpha) f(\alpha) + \left[\ln\frac{\alpha_0 - 1}{\alpha - 1} - \alpha\right]\ln\frac{\alpha_0}{\alpha_0 - 1} + \alpha\ln\frac{\alpha}{\alpha - 1} + h_1(\alpha) = E_0, \tag{29}$$

$$h_1(\alpha) = \frac{\ln^2(\alpha_0 - 1) - \ln^2(\alpha - 1)}{2} - \int\limits_{\alpha}^{\alpha_0}\frac{\ln\alpha'}{\alpha' - 1}\,d\alpha',$$

$$f(\alpha) = \frac{\alpha_0 - 1}{\alpha_0 - \alpha}\ln\frac{\alpha_0 - 1}{\alpha - 1} + \frac{\alpha_0 - \alpha}{2} - 1.$$

Since we are interested in the final state of the medium behind the shock front, we also use Eq. (27) with $\delta = 0$. Eliminating k from these equations, we find the implicit dependence $\alpha_* = \alpha_*(E_0, \alpha_0)$, where α_* is the point on the shock adiabat corresponding to the start of melting.

Results of the numerical solution of Eqs. (27) and (29) are given in Fig. 2, where curves 2 and 4-7 illustrate the dependence $\alpha_*(\alpha_0)$ for the following values of the parameter E_0: 2) 2.3; 4) 3.5; 5) 6; 6) 10; and 7) 18. In the first approximation these dependences are close to the straight lines $\alpha_* - 1 = m(\alpha_0 - 1)$, where it is convenient to approximate the coefficient m by the dependence $m = \exp(-E_0^{0.6})$ in the range of E_0 values under discussion. From this we find, using (27), an approximate formula for the critical shock wave amplitude $P_* = (2Y/3)\ln[\alpha_*/(\alpha_* - 1)]$ at which melting starts [10]:

$$\frac{3P_*}{2Y} = \ln\left[1 + \frac{\exp(E_0^{0,6})}{\alpha_0 - 1}\right]. \tag{30}$$

For small porosities this relationship reduces to the form

$$3P_{*}/(2Y) = E_0^{0.6} - \ln(\alpha_0 - 1).$$

The quantity E_0 depends most significantly on the strength parameter of the medium Y. For example, for tempered steel (c_V = 0.11 cal/g·deg, $T_{*} - T_0$ = 1510°K, ε_0 = 49 cal/g, Y = 1.88 GPa) we obtain that $E_0 \approx 5.9$ [8]. One should also take account of the fact that the experimental value of the coefficient 2Y/3 is larger than the static value according to the measurement data for the shock adiabats of porous metals [2, 11]. The parameter E_0 can be small at a high initial temperature T_0.

The condition $R \gg 1$ is valid for the majority of metals. Thus for iron with $a_0 = 10^{-2}$ cm and $\eta = 10^5$ P [12] we find that $R \approx 42$. The value of the parameter η is usually small (for metals $n \sim 10^{-6}$–10^{-7}), and its effect is insignificant.

At the point $\alpha = \alpha_{*}$ the shock adiabat of a porous material in the variables P and V (here $V = \alpha/\rho_s$ is the specific volume of the medium) experiences a discontinuity caused by the onset of melting. This discontinuity is not related to the change in volume associated with the phase transition, since one can neglect it in this case, but is caused by a decrease in the shear strength of the medium upon shock compression due to the formation of local molten zones next to the pores [10].

In order to determine the shock adiabat of a medium when $\alpha < \alpha_{*}$, it is necessary to know the size of the melt relative to the volume in the final state. Eliminating the parameter k from (26) and (28), we find with the help of (27) an integral equation which relates the quantities α and δ(α) behind the shock wave front:

$$E_0 = \ln\frac{\delta+\alpha_0-1}{\delta+\alpha-1} + \frac{3}{2k^2(\alpha,\delta)}\int\limits_{\alpha}^{\alpha_0}\frac{\alpha'(\alpha'-1)(\delta'+\alpha'-1)F(\alpha',\delta',k^2(\alpha,\delta))}{[(1-\delta')(\alpha'-1)+n\delta'\alpha'][\delta+\alpha'-1]^2}\,d\alpha', \tag{31}$$

$$k^2(\alpha,\delta) = \frac{3(\alpha_{*}-\alpha)}{2\alpha_0^2}\left\{\ln\frac{\alpha(\alpha_0-1)}{\alpha_0(\delta+\alpha-1)}\right\}^{-1}.$$

This equation is satisfied identically at $\alpha = \alpha_{*}$ and $\delta = 0$. We will expand (31) near α_{*} in $(\alpha - \alpha_{*})$, assuming that the derivative $d\delta/d\alpha(\alpha_{*})$ is finite. Taking account of the fact that g does not depend upon δ' in the first approximation, since an expansion in δ' under the integral sign gives corrections $\sim\delta(\alpha_{*} - \alpha)$ of second-order smallness, we obtain the relationship

$$\delta = Q_1(\alpha_{*}-\alpha), \tag{32}$$

$$Q_1 = [H(\alpha_{*})]^{-1}\left\{\frac{1}{\alpha_{*}-1} + \left[\frac{\alpha_0-\alpha_{*}}{\alpha_{*}(\alpha_{*}-1)} - \ln\frac{\alpha_{*}(\alpha_0-1)}{\alpha_0(\alpha_{*}-1)}\right]\frac{f(\alpha_{*})}{(\alpha_0-\alpha_{*})}\right\},$$

$$H(\alpha) = 2\left\{\left[\frac{2-\alpha}{\alpha-1} + \frac{\alpha_0-2}{\alpha_0-\alpha}\ln\frac{\alpha_0-1}{\alpha-1}\right]\ln\frac{\alpha(\alpha_0-1)}{\alpha_0(\alpha-1)}\right.$$
$$+\left[\frac{\alpha_0-\alpha}{(\alpha_0-1)(\alpha-1)} + \ln\frac{\alpha_0-1}{\alpha-1}\right]\ln\frac{\alpha_0}{\alpha_0-1} + \frac{\alpha_0}{\alpha_0-1}\ln\frac{\alpha_0}{\alpha_0-1}$$
$$\left.-\frac{\alpha}{\alpha-1}\ln\frac{\alpha}{\alpha-1} + h_1(\alpha)\right\} + \frac{f(\alpha)}{\alpha-1} + \frac{3(\alpha_0-\alpha)}{(\alpha_0-1)(\alpha-1)}.$$

The course of the shock adiabat beyond the point of discontinuity ($\alpha < \alpha_{*}$) is described by the equation $3P/(2Y) = \ln[\alpha/(\delta + \alpha - 1)]$. The inclination angle γ of the tangent to the shock adiabat at the point $\alpha = \alpha_{*}$ is given by the expression $\gamma = (1 - \alpha_{*}Q_1)/[\alpha_{*}(\alpha_{*} - 1)]$, and the inclination angle of the ray joining the initial and final states (the Rayleigh line) is equal to $\beta = (\alpha_0 - \alpha_{*})^{-1}\ln\{\alpha_{*}(\alpha_0(\alpha_{*} - 1)]\}$. The calculations show that for values of $\alpha_{*} = \alpha_{*}(\alpha_0, E_0)$ lying above line 3 in Fig. 2 the condition $\gamma < \beta$ is satisfied. In this case the medium possesses anomalous properties near the discontinuity of the shock adiabat: The onset of melting in such a porous material leads to the formation of a double-wave structure of the shock wave, and melting occurs in the second wave. Line 3 is close to a straight line; therefore the peculiarities of behavior of the medium are determined mainly by the value of E_0, which specifies the inclination angles of the curves $\alpha_{*}(\alpha_0, E_0)$ in Fig. 2. Thus for values $E_0 \leqslant 3.2$ the medium exhibits anomalous behavior for any value of α_0.

The rate of volume deformation g_2 in the second shock wave, which we will also assume to be steady, is described similarly to expression (28) for g (on the assumption $R \gg 1$)

with replacement of α_0 and k by α_* and $k_2 = (Y/\rho_S)^{1/2} U^{-1}$, where U is the velocity of the second wave in the coordinate system (y, t) moving with the medium:

$$g_2 = \frac{d\alpha}{d\xi_2} = \frac{3\alpha F_2(\alpha)(\delta + \alpha - 1)(\alpha - 1)}{4Rk_2[(1-\delta)(\alpha - 1) + n\delta\alpha]}. \tag{33}$$

Here $\xi_2 = (y - Ut)/\alpha_0$ and $F_2(\alpha)$ is determined in the same manner as is the function $F(\alpha)$ in (25), where one should make the replacements indicated above. The position of the melting front in this case is given by the equation

$$E_0 = \ln\frac{\delta + \alpha_0 - 1}{\delta + \alpha - 1} + \frac{2R}{k_*}\int\limits_{\alpha_*}^{\alpha_0} \frac{g(\alpha')\,d\alpha'}{[\delta + \alpha' - 1]^2} + \frac{2R}{k_2}\int\limits_{\alpha}^{\alpha_*} \frac{g_2(\alpha', \delta')\,d\alpha'}{[\delta + \alpha' - 1]^2}, \tag{34}$$

in which the last term gives the increment to the internal energy due to viscous friction in the second wave. The quantity k_*, which is expressible in terms of α_* with the help of (27), corresponds to the critical amplitude P_*. The position of the shock adiabat after the discontinuity ($\alpha < \alpha_*$) is obtained from a combined consideration of Eqs. (33) and (34), taking account of the fact that $F_2(\alpha) = 0$ in the final state. The relation between the quantities δ and α behind the wave front near the point $\alpha = \alpha_*$ is determined as before by relationship (32), since the third term in (34) gives corrections of second-order smallness. Expanding the right-hand side of Eq. (34) near α_* and using (32), we obtain the following asymptotic dependence, which permits estimating the characteristic scale λ of the second shock wave:

$$\alpha_* - \alpha = c\exp(-\xi_2/\lambda), \quad \lambda = \frac{4\alpha_* R k_2}{(\alpha_* - 1)\left(3 - 2k_2^2\gamma\alpha_*^2\right)}, \tag{35}$$

where c is a constant of integration. It follows from (35) that the minimum value of the wave velocity is determined by the expression $U_{\min}^2 = 2Y\gamma\alpha_*^2/3\rho_s$. In contrast to the first shock wave, whose characteristic size is $\Delta = 2Rk/(k_0^2 - k^2)$ [9], where $k_0^2 = 3(\alpha_0 - 1)/2\alpha_0$, the parameter λ depends significantly on the quantity $\alpha_* = \alpha_*(\alpha_0, E_0)$.

Sections of the shock adiabats near the points of discontinuity are presented in Figs. 3 and 4 for initial porosities $\alpha = 2$ and 1.2; curve 1 depicts the shock adiabat of a porous material without melting taken into account, curves 2 and 5 depict the shock adiabats for the values $E_0 = 6$ and 2.3, and curves 3 and 4 depict the sections of the Rayleigh lines which have passed through the corresponding points of discontinuity.

The effect of thermal conductivity is exhibited in that curves 2 and 5 are more mildly sloping, since the return flow of heat from heated regions facilitates expansion of the melting zones. The decrease of the quantity Y as the temperature in the solid phase increases shows an analogous effect.

Investigation shows (Figs. 3 and 4) that due to local melting effects total closing of the cavities is possible (this statement is valid when gas is completely absent from the pores) for comparatively small amplitudes of the shock waves (of the order of 1 GPa) [10]. Along with the indicated mechanism, collapse of the pores at low pressures can occur due to inertial motion of material towards the cavity centers [6].

Calculations show that the coefficient $\gamma < 0$ for points (α_*, α_0) lying above line 1 in Fig. 2. It follows from an analysis of Eq. (33) that the final steady state in this case is $\alpha = 1$. Line 1 is close to a straight line whose slope corresponds to the value $E_0 \approx 1.3$. Thus the shock adiabats of porous materials have a horizontal section in the P–α diagram at $\alpha < \alpha_*$ when $E_0 \lesssim 1.3$. Such media are unstable with respect to shock compression having an amplitude larger than P_*. Loss of strength due to melting results in the fact that for $P > P_*$ the material behaves similarly to a porous liquid: The pores collapse if the pressure is greater than P_*.

The distinctive features of shock compression associated with the effect of melting on the strength of a porous medium have not been observed experimentally. Calculations show that under normal conditions these effects should appear for low-melting strong materials (E_0 is small).

Let us analyze the variation of the volume of molten material behind the shock wave front as a function of its amplitude, restricting ourselves to a discussion of the case in which one can neglect the dependence of the deformation rate g on δ under the integral sign

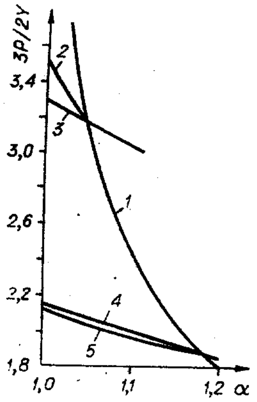

Fig. 3

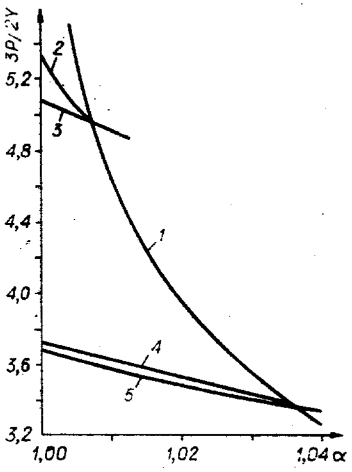

Fig. 4

in (26). We will assume that the medium behind the wave front is continuous and the transition of the material to the final state occurs continuously without formation of a double-wave structure.

Performing the integration in (26) and transforming the result obtained with the help of (24), we obtain the expression

$$3(P-P_0)/(2Y)=[E_0-M(\delta)]L^{-1}(\delta). \tag{36}$$

$$M(\delta)=2\ln\frac{\delta+\alpha_0-1}{\delta\alpha_0}+(2\delta-1)\left\{\ln\frac{\alpha_0}{\alpha_0-1}\ln\frac{\delta}{\alpha_0-1+\delta}+\int\limits_1^{\alpha_0}\frac{\ln\frac{\alpha}{\alpha-1}}{\alpha+\delta-1}d\alpha\right\},$$

$$L(\delta)=\frac{\alpha_0-1}{2}+3\delta-2-\frac{[(2\delta-1)(\delta+\alpha_0-1)-\delta(1-\delta)]}{\alpha_0-1}\ln\frac{\alpha_0+\delta-1}{\delta}.$$

For $\delta \ll 1$ and pressures close to P_*, when incomplete closing of the pores is possible, we find the dependence $\delta(P)$ by making use of relationship (32):

$$\frac{\delta}{Q_1}=\frac{\alpha_*-(\alpha_*-1)\exp[3P/(2Y)]}{1+(Q_1-1)\exp[3P/(2Y)]}. \tag{37}$$

Equations (36) and (37) give the dependence of the relative fraction of melting δ on the wave amplitude P over a wide range of variation of these quantities. The dependences $\delta(P)$ calculated from (36) for porosities $\alpha_0 = 2$ and 1.2 are presented in Figs. 5 and 6; curves 1 and 2 correspond to the values $E_0 = 2.3$ and 6. At the pressure level P_{avg} (lines 1' and 2') corresponding to an increment to the average internal energy in the shock wave $\sim\varepsilon_*$ (the average temperature $\sim T_*$), the relative melt volume is only 0.2-0.3 in all. This peculiarity is associated with the fact that most of the dissipatable energy is concentrated near the pores. The nonuniformity of the thermal energy distribution depends on the initial porosity. Thus in the limit $\alpha_0 \to 1$ expression (36) gives $P = 2E_0/(\alpha_0 - 1) \sim P_{avg}$.

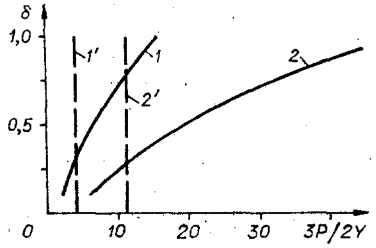

Fig. 5

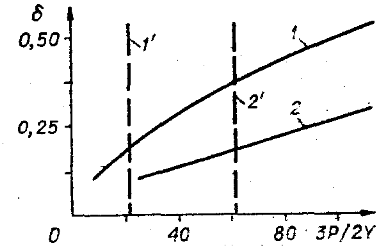

Fig. 6

We will estimate the range of variation of the parameters in which the effect of thermal conductivity is unimportant. Taking account of the fact that most of the energy is concentrated in the vicinity of a pore in a sphere of radius $r \sim \sqrt{4\chi t} + a$ ($t = 0$ is the instant at which the shock wave arrives, and χ is the thermal conductivity coefficient), we find that the characteristic time of passage of thermal waves between adjacent pores is $t_T = \alpha_0^2\{[\alpha_0/(\alpha_0 - 1)]^{1/3} - 1\}^2/(4\chi)$. Comparing this value with the time scale of the shock wave $t_W \sim \Delta/D$, we obtain, for example, for iron ($\chi = 0.18$ cm²/sec, $\alpha_0 = 1.2-2$) that these times are comparable when a_0 10^{-3} cm, and t_T t_W for $a_0 \geqslant 10^{-2}$ cm. The last inequality is also valid for soil-type media having a low thermal conductivity.

One can estimate the maximum temperature in the shock wave by using expression (36) with a variable left-hand side for $\delta = 0$. Thus, calculations of the temperatures upon shock compression of porous Al ($\alpha_0 = 2$, $P = 5$ GPa) give values of the average temperature $T_{avg} = 1350$°K and $T_{max} = 1.4 \cdot 10^4$ °K, which is in agreement with the experimental data [4] for the very same parameter values for which $T_{avg} \sim 1500$°K, and the measured brightness temperature when the shock wave reached a surface of compressed Al powder was $\sim 10^4$ °K.

As a result of the calculations made, the distribution of the internal energy in the vicinity of cavities upon shock compression of porous bodies has been obtained. It has been shown that in the pressure range $P \gg Y$ energy dissipation occurs mainly due to viscous friction; in the case of low stresses in the wave ($P \sim Y$) plasticity makes the main contribution to energy dissipation.

The nature of energy accumulation in the vicinity of an inhomogeneity has been analyzed. Energy dissipation occurs most strongly on the cavity surfaces. When the pores collapse, the specific dissipated energy on pore boundaries has logarithmic-type singularities due to plasticity and power-law type $(\alpha - 1)^{-5/6}$ due to viscous friction.

It has been shown that a noticeable difference in behavior of the shock adiabats of materials of different initial porosity starts to appear at a pressure level corresponding to complete closing of the pores. Thus, for highly porous materials ($\alpha_0 > 1 + 2/\Gamma$) the shock adiabats have a turning point which separates the section of anomalous behavior of the material.

The analysis performed in this paper has permitted establishing the distinctive features of shock compression of porous media produced by local melting of the material in the vicinity of inhomogeneities.

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EFFECT OF TEMPERATURE ON SPALL OF POLYMERIC MATERIALS

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UDC 539.4:620.171.3

The results of an investigation of spall of some polymeric materials are given in [1-9]. The largest number of experimental results have been obtained for Plexiglas [1-6]; spall of polycarbonate has been investigated in detail [7, 8]. Concerning the question of the effect of temperature on spall, it has been experimentally investigated only for some metals (see, for example, [10, 11]), and with respect to polymeric materials one can mention only [12], in which the rate of growth of microdefects of Plexiglas at room and increased temperatures was estimated using the light-scattering method.

The results of an experimental investigation of spall of eight polymeric materials are given in this paper. Identical loading conditions and the use of methods of preliminary heating and cooling of the samples permit obtaining a realistic comparative picture of spall of the materials investigated and determining the nature of the effect of temperature on their spall resistance.

The layout of the experimental setup for investigation of the effect of temperature on spall of materials has been given in [11], and the testing scheme is shown in Fig. 1, where the numbers denote the following: 1) aluminum striker, 2) copper screen, 3) sample mounting, and 4) sample of the material being investigated. Samples 4 mm thick and 40 mm in diameter were loaded by the impact on a copper screen 12 mm thick of a plate made out of AMts aluminum alloy 4 mm thick, whose velocity w was specified with an accuracy of about 3%. Heating of the samples was accomplished through the copper screen by means of a Nichrome ribbon heater. The heating time did not exceed 10 min, and the temperature was recorded with the help of a Chromel–Copel thermocouple. Cooling to a temperature of –196°C was accomplished by submerging the screen with the sample in a vessel containing liquid nitrogen.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 143-150, January-February, 1982. Original article submitted November 20, 1980.

0021-8944/82/2301-0134$07.50